

Chromomagnetic Instability and Gluonic Phase*

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We briefly report on a recent development in studies of a phase with vector condensates of gluons (gluonic phase) in dense two-flavor quark matter.

1. Introduction

It has been suggested that quark matter might exist inside central regions of compact stars.¹ At sufficiently high baryon density, cold quark matter is expected to be in a color superconducting (CSC) state.² This is one of the reasons why the color superconductivity has been intensively studied.³

Bulk matter in the compact stars must be in β -equilibrium and be electrically and color neutral. The electric and color neutrality conditions play a crucial role in the dynamics of quark pairing.⁴ In addition, the strange quark mass cannot be neglected in moderately dense quark matter. Then a mismatch $\delta\mu$ between the Fermi surfaces of the pairing quarks is induced.

As the mismatch $\delta\mu$ grows, the CSC state tends to be destroyed. However, the dynamics is not yet solved completely. It is one of the central issues in this field to reveal the phase structure.

The problem is that the (gapped/gapless) two-flavor color superconducting phase (2SC/g2SC) suffers from the chromomagnetic instability connected with tachyonic Meissner screening masses of gluons.⁵ While the Meissner mass for the 8th gluon is imaginary in the g2SC phase $\delta\mu > \Delta$, where Δ is a diquark gap, the chromomagnetic instability for the 4-7th gluons appears in $\delta\mu > \Delta/\sqrt{2}$. Such a chromomagnetic instability has been found also in the gapless color-flavor locked (gCFL) phase.⁶⁻⁸

Since the Meissner masses are defined at zero momentum and thus *not*

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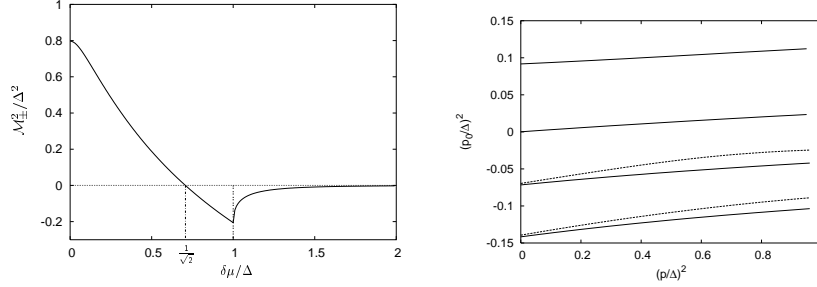


Fig. 1. Mass-gap-squared of the magnetic and electric plasmons for the 4-7th gluons (l.h.s.) and dispersion relations for the 4-7th magnetic plasmons (r.h.s.). In the r.h.s., the bold lines are for $\delta\mu/\Delta = 0.6, 1/\sqrt{2}, 0.8$, and 0.9 , (2SC region) from top to bottom, while the dashed lines are for $\delta\mu/\Delta = 1.065$ and 1.009 , (g2SC region) from top to bottom.

pole ones, the physical origin of the chromomagnetic instability is not obvious. We then study the spectrum of plasmons with nonzero energy and momenta.⁹ We also analyze the dispersion relations of the diquark fields.¹⁰ Then we will find that certain instabilities appear both in $\delta\mu > \Delta/\sqrt{2}$ and $\delta\mu > \Delta$, corresponding to the chromomagnetic instability.

How can we resolve the problem? Actually, numbers of ideas have been proposed by several authors.^{11–16}

As a candidate of the genuine vacuum, we introduce a phase with vectorial gluon condensates (gluonic phase).¹⁵ We also show that the single plane wave Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) phase^{11,17,18} suffers from a chromomagnetic instability.¹⁹

2. Spectra of the plasmons and the diquark excitations

Let us study two point functions of gluons and the diquark fields.

First, we analyze the polarization functions of gluons.⁹

For the 4-7th gluons, we find that the mass gaps for magnetic and electric modes coincide. We depict the results in the l.h.s. of Fig. 1. In the 4-7th channel, there exist the light plasmons in the whole region of $\delta\mu/\Delta$. While in $\delta\mu < \Delta/\sqrt{2}$ the light plasmons have the positive mass-gap-squared, $0 < \mathcal{M}_\pm^2 \lesssim \Delta^2$, in $\delta\mu > \Delta/\sqrt{2}$ the plasmons become tachyons with $\mathcal{M}_\pm^2 < 0$. It is noticeable that the instability occurs *both* for the magnetic and electric modes. This is essentially different from the chromomagnetic instability where the Debye mass for the electric mode remains real.⁵

The dispersion relations for the magnetic and electric modes do not coincide in general. We plot the dispersion relations of the magnetic modes

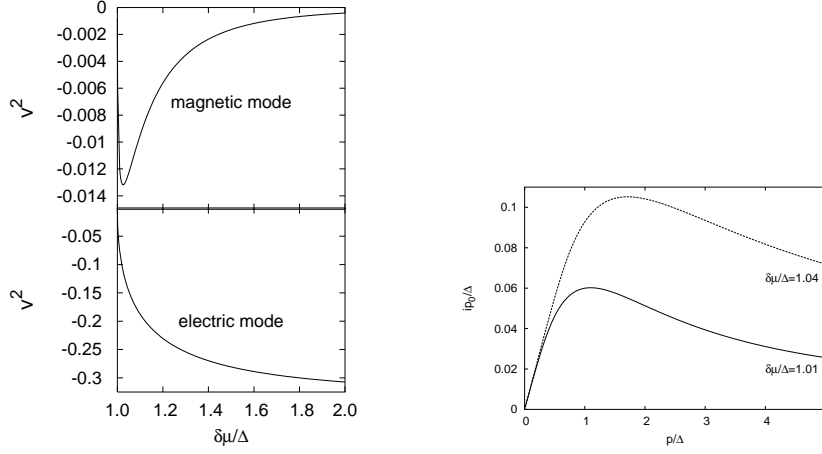


Fig. 2. Velocity squared for gapless tachyons in the magnetic and electric modes of the 8th channel (l.h.s.) and dispersion relations for the gapless magnetic tachyon (r.h.s.).

for the 4-7th channels in the r.h.s. of Fig. 1. The dispersion relations qualitatively read $p_0^2 = m^2 + v^2 p^2$, ($p \equiv |\vec{p}|$) with $m^2 > 0$ for $\delta\mu/\Delta < 1/\sqrt{2}$ and $m^2 < 0$ for $\delta\mu/\Delta > 1/\sqrt{2}$. The velocity v is always real and less than 1.

For the 8th channel, there is no light plasmon in $\delta\mu < \Delta$. On the other hand, in the g2SC region $\delta\mu > \Delta$ there appear gapless tachyons having the dispersion relation $p_0^2 = v^2 p^2$, ($p \simeq 0$) with $v^2 < 0$, as shown in the l.h.s. of Fig. 2. This instability occurs *both* for the magnetic and electric modes, so that it also differs from the chromomagnetic instability in the 8th channel.⁵ For the dispersion relations of the magnetic mode, see the r.h.s. of Fig. 2.

The appearance of the gapless tachyons in the 8th channel seems to be counter-intuitive, because the Meissner mass squared is nonzero and negative in the g2SC phase.⁵ The origin is a singular dependence on p_0/p in the polarization function of the 8th gluon.⁹

Let us turn to discuss the two point functions of the diquark fields in the framework of a Nambu-Jona-Lasinio (NJL) model,^a based on Ref. 10.

While we omit the results for $\Phi^{r,g}$, we do not those for ϕ_{b2} for clarity.

We depict the dispersion relations for ϕ_{b1} (dashed curve) and ϕ_{b2} (bold

^a As usual, we take the anti-blue direction for the vacuum expectation value (VEV) of the diquark fields Φ^α , ($\alpha = r, g, b$), i.e., $\langle \Phi^b \rangle = \Delta$. If the model is gauged, the anti-red and green diquark fields $\Phi^{r,g}$ are eaten by the 4-7th gluons, while the imaginary part ϕ_{b2} of the anti-blue diquark field Φ^b becomes the longitudinal mode of the 8th gluon. Only the real part ϕ_{b1} is left as a physical mode.

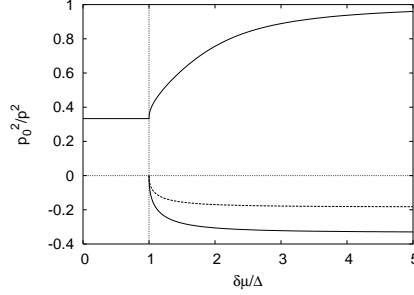


Fig. 3. Velocity squared for the anti-blue diquark field Φ^b . The bold dashed and solid curves are for ϕ_{b1} and ϕ_{b2} , respectively, where ϕ_{b1} and ϕ_{b2} are the real and imaginary parts of Φ^b , respectively. We ignored heavy excitations for ϕ_{b1} .

curves) in Fig. 3. We find that the velocity squared for ϕ_{b2} is $v^2 = 1/3$ in $\delta\mu < \Delta$, while there are two branches in $\delta\mu > \Delta$; one is, say, an ultra-relativistic branch with $1/3 < v^2 < 1$ and the other is a tachyonic one with $v^2 < 0$. For ϕ_{b1} , we ignore a heavy mode,^{20–22} because it should be irrelevant to any instabilities. Actually, there is no light excitation of ϕ_{b1} in $\delta\mu < \Delta$. Surprising is that even for ϕ_{b1} a gapless tachyon with $v^2 < 0$ emerges in $\delta\mu > \Delta$,^b as shown in the dashed curve in Fig. 3. (Another branch for ϕ_{b1} corresponding to a heavy mode may exist in $\delta\mu > \Delta$.²²)

We here comment that a singular dependence on p_0/p in the two point function of Φ^b causes the peculiar behaviors of the dispersion relations for ϕ_{b1} and ϕ_{b2} in $\delta\mu > \Delta$.¹⁰

3. Gluonic phase

What do the instabilities imply? Our answer is the existence of the vectorial gluon condensates (gluonic phase).

In order to clarify the problem, let us consider the $SU(2)_c$ decomposition owing to the symmetry breaking $SU(3)_c \rightarrow SU(2)_c$ in the presence of the diquark condensate Δ . The adjoint representation of $SU(3)_c$, i.e., the gluon field A_μ^a , ($a = 1, 2, \dots, 8$), is decomposed into $\mathbf{3} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{1}$, namely, $\{A_\mu^a\} = (A_\mu^1, A_\mu^2, A_\mu^3) \oplus \phi_\mu \oplus \phi_\mu^* \oplus A_\mu^8$. Here we introduced the complex

^b A similar instability is also discussed in Refs. 23,24.

$SU(2)_c$ doublets of the vectorial “matter” fields,

$$\phi_\mu \equiv \begin{pmatrix} \phi_\mu^r \\ \phi_\mu^g \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^4 - iA_\mu^5 \\ A_\mu^6 - iA_\mu^7 \end{pmatrix}, \quad \phi_\mu^* \equiv \begin{pmatrix} \phi_\mu^{*r} \\ \phi_\mu^{*g} \end{pmatrix}. \quad (1)$$

In the gluonic phase, the spatial component of ϕ_μ develops the VEV, so that the role is quite similar to the Higgs doublet in the standard model.

Let us assume nonvanishing VEVs as

$$B \equiv g\langle A_z^6 \rangle, \quad C \equiv g\langle A_z^1 \rangle, \quad D \equiv \mu_3 = g\langle A_0^3 \rangle, \quad (2)$$

where C is required for consistency with the Ginzburg-Landau (GL) approach which we will employ. For a more general ansatz, see Ref.25.

Neglecting irrelevant terms, we obtain a reduced GL potential,¹⁵

$$\tilde{V}_{\text{eff}} = V_\Delta + \frac{1}{2}M_B^2 B^2 + TDB^2 + \frac{1}{2}\lambda_{BC}B^2C^2 + \frac{1}{2}\lambda_{CD}C^2D^2, \quad (3)$$

where V_Δ is the 2SC part and the parameter M_B^2 is expressed through the Meissner mass. The negative M_B^2 essentially dictates a nonvanishing gluon condensate $B \neq 0$. The coefficients T , λ_{BC} and λ_{CD} are calculated in the fermion one-loop approximation.¹⁵ We then find that λ_{CD} is definitely negative and $\lambda_{BC} > 0$ in the vicinity of the critical point $\delta\mu \approx \Delta/\sqrt{2}$. The free energy at the stationary point is found as

$$\tilde{V}_{\text{eff}} = V_\Delta - \frac{(-M_B^2)^3}{54T^2} \left(-\frac{\lambda_{CD}}{\lambda_{BC}} \right) < V_\Delta. \quad (4)$$

Therefore the gluonic vacuum is stabler than the 2SC one. We can check also that the solution corresponds to a minimum.

It is noticeable that the above gluonic solution describes *non-Abelian* constant chromoelectric fields, $F_{0j}^a \neq 0$. In this sense, the gluonic phase enjoys a non-Abelian nature. We emphasize that while an Abelian constant electric field always leads to an instability, non-Abelian one does not in many cases.²⁶ The difference seems to be connected with the fact that a constant Abelian electric field is derived only from a vector potential depending on spatial and/or time coordinates, while a constant non-Abelian chromoelectric field can be expressed through constant vector potentials owing to nonzero commutators. Thus energy and momentum can be left as good quantum numbers in the non-Abelian case.

In the gluonic phase, both the rotational $SO(3)$ and the electromagnetic $U(1)$ symmetries are spontaneously broken down. Therefore, this phase describes an anisotropic medium with the color and electromagnetic Meissner effects. There also exist exotic hadrons in the medium.

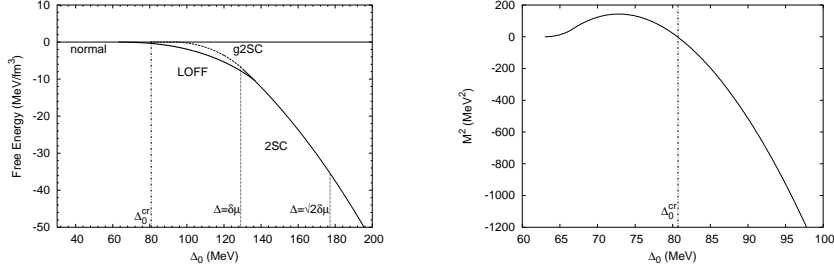


Fig. 4. Free energy of the neutral LOFF state [bold line], and the neutral 2SC/g2SC state [solid/dashed line] (l.h.s.) and Meissner mass squared M^2 for the 4-7th gluons in the LOFF phase (r.h.s.). In the l.h.s. the free energy of the neutral normal state is chosen as a reference point. In the r.h.s. we took the QCD coupling constant $\alpha_s = 1$.

4. The single plane wave LOFF state and its instability

In order to demonstrate how the neutrality conditions work and dramatically change the situation, we analyze the single plane wave LOFF state^c.¹⁹

For a numerical analysis, we fix the quark chemical potential $\mu=400$ MeV and the cutoff $\Lambda = 653.3$ MeV, and vary the value of Δ_0 , which is the 2SC gap parameter at $\delta\mu = 0$. We show the free energy differences in the l.h.s. of Fig. 4 (the reference point is the free energy of the normal phase with $\Delta = 0$). The results are not sensitive to the choice of μ ($= 300$ – 500 MeV).

One can see that the neutral LOFF phase is energetically stabler than the neutral normal phase and the neutral g2SC/2SC one in the whole region of the g2SC plus a narrow region of the 2SC near the edge, i.e., $63 \text{ MeV} < \Delta_0 < 136 \text{ MeV}$. Since the chromomagnetic instability in the 2SC phase occurs in the region $\Delta < \sqrt{2}\delta\mu$, which corresponds to $\Delta_0 = 177 \text{ MeV}$ in the l.h.s. of Fig. 4, the neutral LOFF solution cannot cure the instability.

Furthermore, by applying the Meissner mass formula in the second paper in Ref.18 to our LOFF solution, we find that the neutral LOFF state itself suffers from a chromomagnetic instability in $\Delta_0 > \Delta_0^{cr} = 81 \text{ MeV}$ and thereby it should not be the genuine ground state. (See the r.h.s. of Fig. 4.)

^c The order parameter of the single plane wave LOFF state has the form $\langle \Phi^b(x) \rangle = \Delta e^{2i\vec{q} \cdot \vec{x}}$ with a constant phase vector \vec{q} , instead of $\langle \Phi^b \rangle = \Delta$ in the 2SC/g2SC phase. This phase is gauge equivalent to the phase with $\langle \Phi^b \rangle = \Delta$ and $\langle \vec{A}^8 \rangle \neq 0$.^{15,19}

5. Summary and discussions

We showed that in the 2SC/g2SC phase there appear several instabilities other than the chromomagnetic one.

We introduced the gluonic phase to resolve the instabilities. We also found that the neutral LOFF state is not free from the instabilities. It indicates that the gluonic phase is relevant for curing the problem.^d

It is worthwhile to examine the multiple plane-wave 2SC LOFF state.²⁸ It would be also interesting to figure out whether or not a phase with vectorial gluon condensates exists in three-flavor quark matter.

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^d See also a numerical approach of the gluonic phase.²⁷

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